Imperial College London

Topic 3

Resistors & Resistor Circuits

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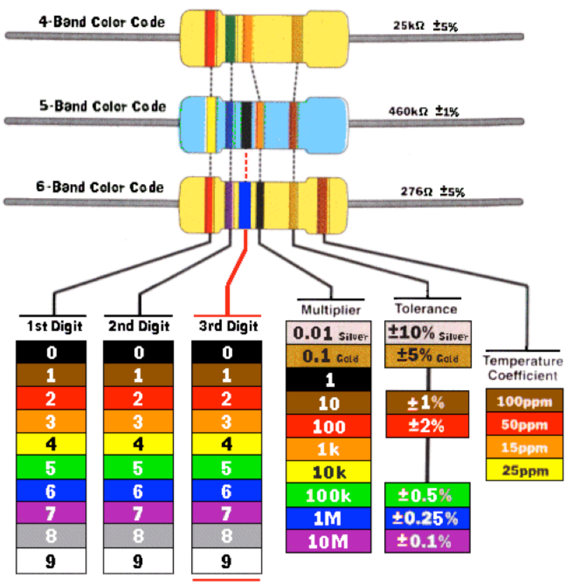
In this lecture, we will learn about resistors and resistor networks, and how to simply them.

Resistor parameters and identification

- Resistors are usually colour coded with their values and other characteristics as shown here.
- They also come in different tolerances (e.g. ±0.1% to ±10%).
- Other important parameters are:
  - Power rating (in Watts)
  - Temperature coefficient in parts per million (ppm) per degree C
  - Stability over time (also in ppm)
  - Inductance (don't worry about this for now)
- Resistors can be made of different materials: carbon composite (most common), enamel, ceramic etc.



A resistor is characterised by a number of parameters:

1. Its nominal value;
2. Its tolerance or accuracy (e.g. ±5%);
3. Its power rating (i.e. maximum power that it can dissipate);
4. Its temperature coefficient (how much the resistance vary with temperature);
5. Its stability (i.e. how much it changes over time);
6. Its self inductance (something we don't worry about unless you are using resistors at very very high frequencies).

These characteristics are often shown on the resistor itself as a colour code.

The colour code is as shown above. (The printed notes are not in colour. You can download the PDF file from the course webpage, which will be shown in full glorious colours.)

Consider the top resistor. It has four bands, and the band colours are:

RED, GREEN, ORANGE, a gap, BROWN

The first two colour bands are the first two digits of the resistance, i.e. RED = 2, GREEN = 5. The third band in this case is the multiplier. ORANGE = 10^3 or 1k. The gap is always there to separate value bands from tolerance band. BROWN =±5%.

Resistor – Preferred values

- In theory, resistor values is a continuous quantity with infinite different values.
- In reality, resistor as a component exists within some tolerance (say, ±5% is common)
- Therefore there is NO reason to provide more than selected number of different resistor values for a given tolerance.
- The standard "preferred values" for resistors are given in this table for ±5% (most common), ±10% and ±20%, respectively designated as the E24, E12, E6 series.
- For example, if you need a 31.3kΩ resistor with tolerance of ±10%, you could use a 30kΩ E24 resistor (±5%) instead and still stay within the allowable tolerance.
- Therefore, when computing solutions resistor values for electronic circuits, it is silly to use precision with many digits.

Resistor Values

E6 (20%)	E12 (10%)	E24 (5%)
10	10	10
		11
	12	12
		13
15	15	15
		16
	18	18
		20
22	22	22
		24
	27	27
		30
33	33	33
		36
	39	39
		43
47	47	47
		51
	56	56
		62
68	68	68
		75
	82	82
		91

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Since resistors have tolerances, it is not necessary normal even sensible to provide resistors of ALL values. Let us suppose you have a 1kΩ resistor with a tolerance of 10%. This resistor could vary from 900Ω to 1.1kΩ in value. You want to guarantee that another resistor with lower nominal value is always lower in resistance. Therefore it does not make sense to provide any resistance with a value above 820Ω, say 850Ω. This is because 850Ω at 10% would give you a range of 765Ω to 935Ω, which would be higher than the lowest value of the 1k resistor!

Therefore in industry, only selected values (known as Preferred Values) of resistors are made, dependent on the tolerance. Shown here are the ±20%, ±10% and ±5% resistors values in a decade range. They are called E6, E12 and E24 respectively because there are 6, 12 and 24 values in each decade (similar to musical nodes).

GOOD ENGINNERING PRACTICE: you can see that since in engineering design, we always have to consider tolerance, and even the humble resistor only exists in defined values, it does not make sense to use precision in your solutions having many digits.

In our laboratory, we will be mostly using the E24 series of resistors at ±5% tolerance.

Units and Multipliers

Quantity	Letter	Unit	Symbol
Charge	Q	Coulomb	C
Conductance	G	Siemens	S
Current	I	Amp	A
Energy	W	Joule	J
Potential	V	Volt	V
Power	P	Watt	W
Resistance	R	Ohm	Ω

Value	Prefix	Symbol
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

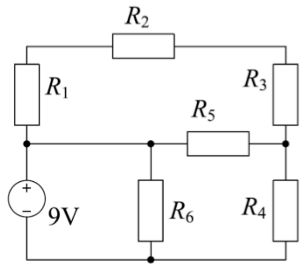
Value	Prefix	Symbol
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Here are the common quantities used in electrical engineering, their units and symbolic representations for the units.

Furthermore, we do not generally use all decades for multipliers (say of resistors), but the multipliers are in steps of THREE decade.

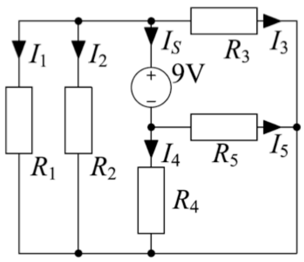
Series and Parallel

Series: Components that are connected in a chain so that the same current flows through each one are said to be *in series*.



- R_1, R_2, R_3 are in series and the same current always flows through each.
- Within the chain, each internal node connects to only two branches.
- R_3 and R_4 are not in series and do not necessarily have the same current.

Parallel: Components that are connected to the same pair of nodes are said to be in parallel .



- R_1, R_2, R_3 are in parallel and the same voltage is across each resistor (even though R_3 is not close to the others).
- R_4 and R_5 are also in parallel.

P52-53

Circuits can be simplified by combining components. In the top circuit, resisters R1, R2 and R3 are connected in series. It can be replaced by ONE resistor R_T = R1 + R2 + R3. with series resistors, the SAME CURRENT flows through all of them.

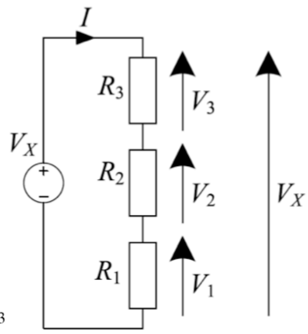
Note that in this circuit, R3 and R4 are NOT in series because they may not have the same current.

Resistors can be connected in parallel. R1, R2 and R3 share the same two nodes, and therefore they have the SAME VOLTAGE across them. R4 and R5 are also in parallel, but only to each other.

Series Resistors: Voltage Divider

$$V_x = V_1 + V_2 + V_3$$
$$= I\,R_1 + I\,R_2 + I\,R_3$$
$$= I\,(R_1 + R_2 + R_3)$$

$$\frac{V_1}{V_x} = \frac{I\,R_1}{I\,(R_1 + R_2 + R_3)} = \frac{R_1}{R_1 + R_2 + R_3} = \frac{R_1}{R_T}$$



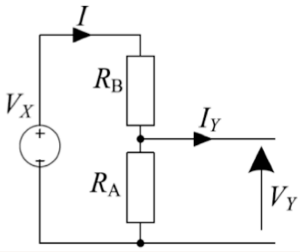
Where R_T is the total resistance of the chain $R_T = R_1 + R_2 + R_3$

V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$

Approximate Voltage Divider:

If $I_Y = 0$, then $V_Y = \frac{R_A}{R_A+R_B} V_X$.

If $I_Y \ll I$, then $V_Y \approx \frac{R_A}{R_A+R_B} V_X$.



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Series connected resistors have the same current through them. Since V = IR (Ohm's law), the resistors DIVIDE the voltage in the same ratio as the resistances. Consider the voltage V1, the simple mathematics shown here shows that V1 / Vx = the ratio of R1 to the total resistance R_T. Therefore Vx is divided into V1:V2:V3, by R1:R2:R3. The higher the resistance Rx, the higher the voltage across it. This is called a voltage divider.

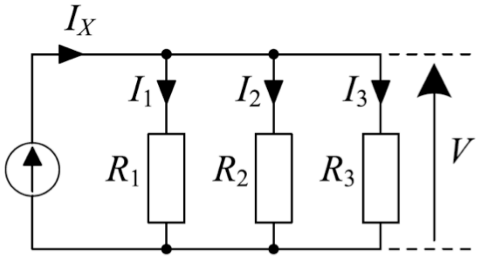
It is very common to use TWO resistors show in the lower diagram to produce a smaller voltage V_Y by dividing V_X as shown. If the output current I_Y is zero (therefore no load connect to V_Y), then the voltage divider is exact. If I_Y is not zero, as long as I_Y is much smaller the I, the current through the series resistor divider circuit, the voltage divider equation is an approximation.

Parallel Resistors: Current Divider

- Parallel resistors all share the same V.

$$I_1 = \frac{V}{R_1} = V\ G_1 \qquad \text{where} \qquad G_1 = \frac{1}{R_1}$$ is the *conductance* of R_1.

$$I_x = I_1 + I_2 + I_3$$
$$= VG_1 + VG_2 + VG_3$$
$$= V(G_1 + G_2 + G_3)$$



$$\frac{I_1}{I_X} = \frac{VG_1}{V(G_1+G_2+G_3)} = \frac{G_1}{G_1+G_2+G_3} = \frac{G_1}{G_P}$$

where $G_P = G_1 + G_2 + G_3$ is the total conductance of the parallel resistors.

I_X is divided into $I_1 : I_2 : I_3$ in the proportions $G_1 : G_2 : G_3$.

While series connected resistors divide a voltage, parallel connected resistors divide current. The current IX is divided amount the three resistors R1, R2 and R3 in the ratio of their CONDUCTANCE (i.e. 1/resistance). The higher the conductance, the higher the current through that resistor.

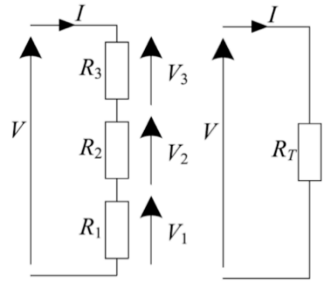
Note that series connected resistors have a total resistance = sum of all resistances that are in series. Similar for parallel resistors, total conductance = sum of all conductances that are in parallel.

Equivalent Resistance: Series

- We know that

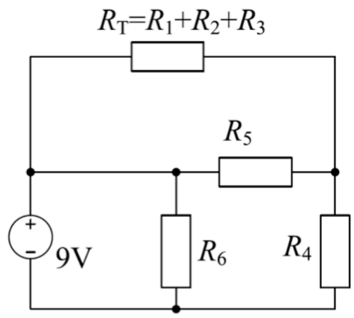
$$V = V_1 + V_2 + V_3 = I\ (R_1 + R_2 + R_3) = I\ R_T$$

- So we can replace the three resistors by a single *equivalent resistor* of value R_T without affecting the relationship between V and I.



- Replacing series resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.

- However the individual voltages V_1, V_2 and V_3 are no longer accessible.



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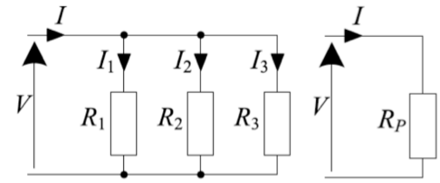
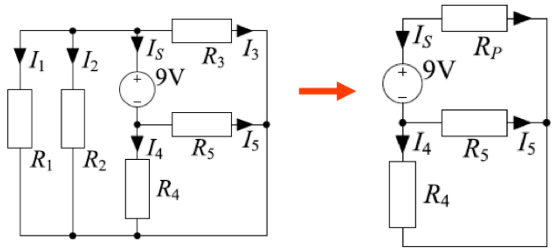
We can replace series connected resistors with an equivalent resistance to simplify the circuit. The top circuit has three resistors R1, R2 and R3. This can be replaced by one resistor with RT = sum of all three resistances. The V-I relationship for the two versions of the circuit are identical. However, the individual voltages V1, V2 and V3 across each of the resistors are no longer accessible.

The circuit at the top right of slide 5 can therefore be simplified to the one shown here.

Equivalent Resistance: Parallel

- Similarly we known that
  $$I = I_1 + I_2 + I_3 = V\ (G_1 + G_2 + G_3) = V\ G_P$$
- So $V = I\ R_P$ where $R_P = \frac{1}{G_F} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$
- We can use a single *equivalent resistor* of resistance R_P without affecting the relationship between V and I.
- Replacing parallel resistors by their equivalent resistor will not affect any of the voltages or currents in the rest of the circuit.
- R_4 and R_5 are also in parallel.
- Much simpler - although none of the original currents I_1, ••••, I_3 are now implicitly specified.





Similarly we can perform the same simplification with parallel connected resistors. In the circuit shown below, R1, R2 and R2 are combined to R_P. We can further combine R4 and R5 because they too are connected in parallel.

Equivalent Resistance: Parallel Formulae

- For parallel resistors $G_P = G_1 + G_2 + G_3$

  or equivalently $R_P = R1 \,||\, R2 \,||\, R3 = \dfrac{1}{1/R_1 + 1/R_2 + 1/R_3}$
- These formulae work for any number of resistors.
- For the special case of two parallel resistors

  $$R_P = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2}$$ ("product over sum")
- If one resistor is a multiple of the other

  Suppose $R_2 = kR_1$, then

  $$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{kR_1^2}{(k+1)R_1} = \frac{k}{k+1} R_1 = (1 - \frac{1}{k+1})\ R_1$$
- Example: $1\ k\Omega \,||\, 99\ k\Omega = \dfrac{99}{100} k\Omega = (1 - \dfrac{1}{100})\ k\Omega$
- **Important**: The equivalent resistance of parallel resistors is always less than any of them.

We often write parallel connected resistors using the double vertical bar symbol. For the special case of TWO parallel resistors, the equivalent resistance is R1*R2/(R1+R2), or product divided by the sum. This formulae is well worth memorizing – you will use this equation often.

Always remember: if you have a resistor R1, and then connect a resistor R2 in parallel with R1, the equivalent resistance is ALWAYS SMALLER than both R1 and R2.

In other words, adding a resistor in series to an existing resistor will INCREASE the total resistance; adding a resistor in parallel to an existing resistor will REDUCE the total resistance.

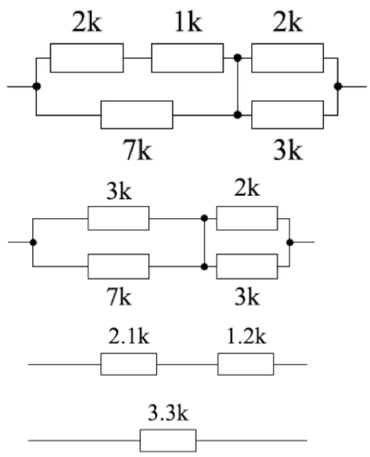
Simplifying Resistor Networks

- Many resistor circuits can be simplified by alternately combining series and parallel resistors.

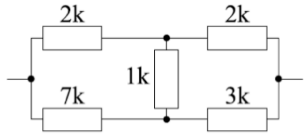
Series: 2 kΩ + 1 kΩ = 3 kΩ

Parallel: 3 kΩ || 7 kΩ = 2.1 kΩ
Parallel: 2 kΩ || 3 kΩ = 1.2 kΩ

Series: 2.1 kΩ + 1.2 kΩ = 3.3 kΩ



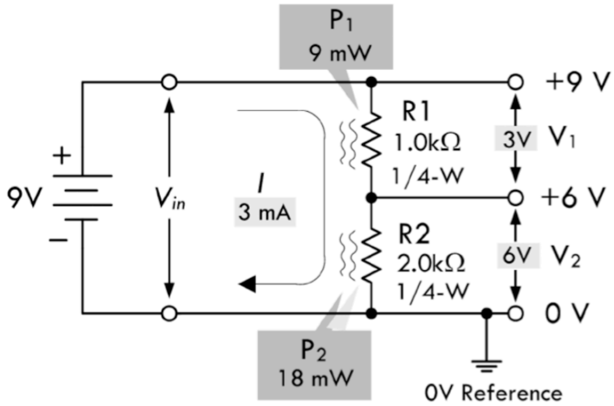
- Sadly this method does not always work: there are no series or parallel resistors here.



Here are a few examples of simplifying resistor networks.

Example of a voltage divider

- Using two resistors R1 and R2, connected to a voltage source 9V, we can produce any voltage between 0V and the battery source voltage of 9V
- In this example, R1 and R2 are connected in series. The total resistance is 3kΩ
- The current I through the two resistors is therefore 9V/3k = 3mA.
- Therefore the voltage across R2 is: $V_2 = I \times R_2 = 6V$
- The voltage across R1 is 3V



- This is called a voltage divider because R1 and R2 effectively divide the 9V into two parts!

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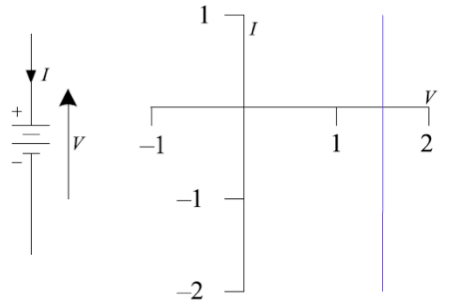
Here is another example of a voltage divider. It divides the 9V battery voltage source into two parts: lower voltage V2 = 6V, and the upper voltage V1 = 3V.

Not that in this case, the diagram also shows the power dissipated by each resistors. As can be seen, the power dissipated by these two resistors are much lower than rated value of 250mW (the same type of resistors that you will be using in the Home Lab Kit).

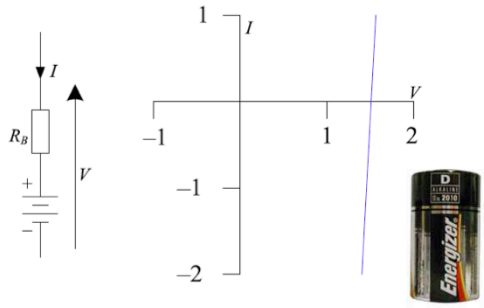
Also remember: if you keep resistors in kW, current in mA, voltage in V, then everything works out fine with Ohm’s Law. In this case, I x R = 3 x 2 = 6V.

Non-ideal Voltage Source

- An ideal battery has a characteristic that is vertical: battery voltage does not vary with current.
- Normally a battery is supplying energy so V and I have opposite signs, so $I \leq 0$.



- An real battery has a characteristic that has a slight positive slope: battery voltage decreases as the (negative) current increases.
- Model this by including a small resistor in series. $V = V_B + IR_B$.
- The equivalent resistance for a battery increases at low temperatures.



In Topic 2, we considered idea voltage DC (direct current) sources such as that found in an ideal battery. The I-V characteristic is a vertical line because no matter what current you draw from the battery, the voltage remains constant.

In practice, this does not happen. In a real battery, if you increase the current you draw from it, the battery voltage decreases. This is the same as having a resistor RB in series with an ideal voltage source. The I-V plot now is a line with a slight positive slope. As you draws more current from the battery, the current is increasingly negative (because current is flowing OUT of the battery and is in the opposite direction of the arrow shown here), the voltage V decreases. RB is often called internal resistance of the battery.

As temperature drops, RB increases. That's why we that cars are harder to start in the winter because the car battery has increased internal resistance.

Summary

- Identify resistor values
- Series and Parallel components
- Voltage and Current Dividers
- Simplifying Resistor Networks
- Battery Internal Resistance